Stringy Quintessence Models
in the Swampland

Max Brinkmann

String Pheno, Liverpool

2022

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

INFN PADOVA
Istituto Nazionale di Fisica Nucleare
Sezione di Padova

UNIVERSITÀ DEGLI STUDI DI PADOVA

Outline

Based on **"Stringy multifield quintessence and the swampland"**
with Michele Cicoli, Giuseppe Dibitetto and Francisco G. Pedro [2206.10649]

- Motivation: swampland vs. observation
- Around the swampland? Multifield quintessence!
- Stringy models I: universal moduli
- Stringy models II: non-universal moduli

Motivation

A cosmological constant problem

- The CC is not a free parameter in quantum gravity.
- In EFT, it is just the vev of the scalar potential.
- A flat potential can sometimes have the effect of a CC (slow-roll).

Motivation

A cosmological constant problem

- The CC is not a free parameter in quantum gravity.
- In EFT, it is just the vev of the scalar potential.
- A flat potential can sometimes have the effect of a CC (slow-roll).

Observations

- CMB: explained by Inflation
- Today: accelerated expansion

$\Rightarrow$ Positive CC, $\Lambda > 0$

Theory

- No dS in parametric control
- Type IIA no-go theorem
- quantum break time
- ...

$\Rightarrow$ (no-)dS conjecture, $\Lambda \leq 0$

dS swampland conjecture and accelerating cosmologies

The (refined) dS conjecture

[Obied/Ooguri/Spodyneiko/Vafa '18; Garg/Krishnan '18; Ooguri/Palti/Shiu/Vafa '18]

The scalar potential V of an EFT coupled to quantum gravity in the UV must respect either

$$|\nabla V| \geq \frac{c}{M_p} \cdot V \qquad \text{or} \qquad \min(\nabla_i \nabla_j V) \leq -\frac{c'}{{M_p}^2} \cdot V$$

with $c, c' > 0$ and $\mathcal{O}(1)$.

dS swampland conjecture and accelerating cosmologies

The (refined) dS conjecture

[Obied/Ooguri/Spodyneiko/Vafa '18; Garg/Krishnan '18; Ooguri/Palti/Shiu/Vafa '18]

The scalar potential V of an EFT coupled to quantum gravity in the UV must respect either

$$|\nabla V| \geq \frac{c}{M_p} \cdot V \qquad \text{or} \qquad \min(\nabla_i \nabla_j V) \leq -\frac{c'}{{M_p}^2} \cdot V$$

with $c, c' > 0$ and $\mathcal{O}(1)$.

Implications for cosmology

- Clearly forbids (meta-)stable dS vacua $\nabla V = 0,\ V > 0$!
- Also flat slow-roll models $\epsilon_V \equiv \frac{1}{2}\left(\frac{\nabla V}{V}\right)^2 \ll 1$ are ruled out.

Multifield cosmology

Alternatives to dS or slow-roll

Accelerated expansion can be achieved with a steep potential, e.g. by **rotating in a curved field space.** [Brown '17; Achúcarro/Palma '18; Cicoli/Dibitetto/Pedro '20]

- Kinetic couplings: can't canonically normalize all fields at once
- Energy dissipates into rotation, slowing down the rolling field

Multifield cosmology

Alternatives to dS or slow-roll

Accelerated expansion can be achieved with a steep potential, e.g. by **rotating in a curved field space.** [Brown '17; Achúcarro/Palma '18; Cicoli/Dibitetto/Pedro '20]

- Kinetic couplings: can't canonically normalize all fields at once
- Energy dissipates into rotation, slowing down the rolling field

Multifield quintessence – from string theory?

- Hard to get many e-folds [Aragam/Chiovoloni/Paban/Rosati/Zavala '21]
- Focus on late-time cosmology instead
- Less e-folds needed, but observational constraints
- 2 field model has structure akin to string moduli

Can ST satisfy the dS conjecture **and** get late-time cosmology right?

Multifield quintessence

Accelerated Expansion

$$\text{Slow-roll:} \qquad \epsilon_H = -\frac{\dot{H}}{H^2}\,, \quad 0 < \epsilon_H < 1$$

Single field case

$$\epsilon_V \equiv \frac{1}{2}\left(\frac{\nabla V}{V}\right)^2 = \epsilon_H\,, \quad \epsilon_V \gtrsim 1 \quad \text{(dS conj.)}$$

The dS conjecture forbids flat potentials needed for slow roll.

Multifield quintessence

Accelerated Expansion

$$\text{Slow-roll:} \quad \epsilon_H = -\frac{\dot{H}}{H^2}\,, \quad 0 < \epsilon_H < 1$$

Single field case

The dS conjecture forbids flat potentials needed for slow roll.

Multifield case

Rotation in moduli space slows down the rolling field.

$$\Rightarrow \quad \epsilon_V \equiv \frac{1}{2}\left(\frac{\nabla V}{V}\right)^2 \neq \epsilon_H \; !$$

Possible to have steeper potentials while accelerating.

The model

Action

Requirements: Gravity, 2 fields, kinetic coupling, scalar potential.

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(\frac{{M_p}^2}{2} R - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2} f(\phi_1)^2 (\partial\phi_2)^2 - V(\phi_1) \right)$$

The model

Action

Requirements: Gravity, 2 fields, kinetic coupling, scalar potential.

$$S = \int \mathrm{d}^4x \, \sqrt{-g} \left(\frac{{M_p}^2}{2} R - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2} f(\phi_1)^2 (\partial\phi_2)^2 - V(\phi_1) \right)$$

Natural situation in String theory

e.g. Kähler moduli in IIB flux vacua: $T = \tau + i\vartheta$, $K = -3\log(T + \overline{T})$

$$\mathcal{L}_{\text{kin}} \sim K_{T\overline{T}} \, \partial T \partial \overline{T} = \frac{3}{4\tau} \left((\partial\tau)^2 + (\partial\vartheta)^2 \right)$$

$$\Rightarrow \quad \phi_1 = \sqrt{3/2} \, \log(\tau) \, , \quad \phi_2 = \vartheta$$

$$f(\phi_1) = \sqrt{3/2} \, e^{\sqrt{3/2}\,\phi_1}$$

$$V(\phi_1) \sim \frac{1}{\tau} = e^{-\sqrt{3/2}\,\phi_1}$$

The model

Friedmann equations

$$H^2 = \frac{1}{6{M_p}^2}\left(\dot{\phi_1}^2 + f^2\dot{\phi_2}^2 + 2V + \rho_{\text{matter}}\right)$$

Defining new variables:

$$x_1 = \dot{\phi}_1 \;\; (\sqrt{6}H\,M_p)^{-1}$$
$$x_2 = f\,\dot{\phi}_2\,(\sqrt{6}H\,M_p)^{-1}$$
$$y_1 = \sqrt{V}\,(\sqrt{3}H\,M_p)^{-1}$$

Cosmological Observables:

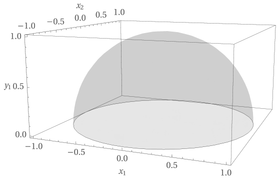
$$\omega_\phi = \frac{{x_1}^2 + {x_2}^2 - {y_1}^2}{{x_1}^2 + {x_2}^2 + {y_1}^2}$$

$$\Omega_\phi = {x_1}^2 + {x_2}^2 + {y_1}^2$$

The model

Friedmann equations

$$H^2 = \frac{1}{6{M_p}^2}\left(\dot{\phi_1}^2 + f^2\dot{\phi_2}^2 + 2V + \rho_{\text{matter}}\right)$$



Cosmological Observables:

$$\omega_\phi = \frac{{x_1}^2 + {x_2}^2 - {y_1}^2}{{x_1}^2 + {x_2}^2 + {y_1}^2}$$

$$\Omega_\phi = {x_1}^2 + {x_2}^2 + {y_1}^2$$

Parameter space

- physical range: $x_1 \in [-1, 1]$, $x_2 \in [-1, 1]$, $y_1 \in [0, 1]$.
- In a flat Universe: $\Omega_{\text{matter}} = 1 - ({x_1}^2 + {x_2}^2 + {y_1}^2) > 0$.

$\Rightarrow$ Physical parameter space is upper half of a 3-ball.

Dynamics

Kinetic coupling

$$k_1 = -M_p \frac{\partial_{\phi_1} f}{f}\,, \qquad k_2 = -M_p \frac{\partial_{\phi_1} V}{V}\,.$$

In general, $k_i = k_i(\phi_i)$.

Evolution as autonomous system:

The dynamics is given in terms of e-folds ($' \equiv d/d(\ln a)$) by

$$x_1{}' = h_1(x_1, x_2, y_1)\,, \quad x_2{}' = h_2(x_1, x_2, y_1)\,, \quad y_1{}' = h_3(x_1, x_2, y_1)\,.$$

Dynamics are completely determined by initial conditions and k_i.

If $k_i = k_i(\phi_i)$, additionally require $\phi_1' = 6x_1$.

Fixed Points

Fixed points $x_1{}' = x_2{}' = y_1{}' = 0$ for constant k_i

	x_1	x_2	y_1	Ω_ϕ	ω_ϕ	stability
$\mathcal{K}_\pm$	± 1	0	0	1	1	unstable
$\mathcal{F}$	0	0	0	0	–	unstable
$\mathcal{S}$	$\frac{\sqrt{3/2}}{k_2}$	0	$\frac{\sqrt{3/2}}{k_2}$	$\frac{3}{k_2^2}$	0	$k_2^2 \geq 3$
$\mathcal{G}$	$\frac{k_2}{\sqrt{6}}$	0	$\sqrt{1-\frac{k_2^2}{6}}$	1	$-1+\frac{k_2^2}{3}$	$k_2 < \sqrt{6}$
$\mathcal{NG}$	$\frac{\sqrt{6}}{(2k_1+k_2)}$	$\frac{\pm\sqrt{k_2^2+2k_2k_1-6}}{2k_1+k_2}$	$\sqrt{\frac{2k_1}{2k_1+k_2}}$	1	$\frac{k_2-2k_1}{k_2+2k_1}$	$k_2 \geq \sqrt{6+k_1^2}-k_1$

Fixed Points

Fixed points $x_1{}' = x_2{}' = y_1{}' = 0$ for constant k_i

	x_1	x_2	y_1	Ω_ϕ	ω_ϕ	stability
$\mathcal{K}_\pm$	± 1	0	0	1	1	unstable
$\mathcal{F}$	0	0	0	0	–	unstable
$\mathcal{S}$	$\frac{\sqrt{3/2}}{k_2}$	0	$\frac{\sqrt{3/2}}{k_2}$	$\frac{3}{k_2^2}$	0	$k_2^2 \geq 3$
$\mathcal{G}$	$\frac{k_2}{\sqrt{6}}$	0	$\sqrt{1 - \frac{k_2^2}{6}}$	1	$-1 + \frac{k_2^2}{3}$	$k_2 < \sqrt{6}$
$\mathcal{NG}$	$\frac{\sqrt{6}}{(2k_1+k_2)}$	$\frac{\pm\sqrt{k_2^2+2k_2k_1-6}}{2k_1+k_2}$	$\sqrt{\frac{2k_1}{2k_1+k_2}}$	1	$\frac{k_2-2k_1}{k_2+2k_1}$	$k_2 \geq \sqrt{6+k_1^2} - k_1$

- The $\mathcal{NG}$ fixed points with $x_2 \neq 0$ exist only for multifield models
- Non-geodesic field trajectories, ϕ_2 dragged along by $\dot{\phi}_1$

Fixed Points

Fixed points $x_1{}' = x_2{}' = y_1{}' = 0$ for constant k_i

	x_1	x_2	y_1	Ω_ϕ	ω_ϕ	stability
$\mathcal{K}_\pm$	± 1	0	0	1	1	unstable
$\mathcal{F}$	0	0	0	0	–	unstable
$\mathcal{S}$	$\frac{\sqrt{3/2}}{k_2}$	0	$\frac{\sqrt{3/2}}{k_2}$	$\frac{3}{k_2^2}$	0	$k_2^2 \geq 3$
$\mathcal{G}$	$\frac{k_2}{\sqrt{6}}$	0	$\sqrt{1 - \frac{k_2^2}{6}}$	1	$-1 + \frac{k_2^2}{3}$	$k_2 < \sqrt{6}$
$\mathcal{NG}$	$\frac{\sqrt{6}}{(2k_1+k_2)}$	$\frac{\pm\sqrt{k_2^2+2k_2k_1-6}}{2k_1+k_2}$	$\sqrt{\frac{2k_1}{2k_1+k_2}}$	1	$\frac{k_2-2k_1}{k_2+2k_1}$	$k_2 \geq \sqrt{6+k_1^2} - k_1$

- The $\mathcal{NG}$ fixed points with $x_2 \neq 0$ exist only for multifield models
- Non-geodesic field trajectories, ϕ_2 dragged along by $\dot{\phi}_1$

- Only $\mathcal{G}$, $\mathcal{NG}$ can be accelerating $\omega_\phi < -1/3$
- But fixed points cannot fit $\Omega_\phi \sim 0.7$. Look for transients!

Approaches to finding transients

Cosmological initial conditions

- Start in the past at phase of matter domination.
- $\Omega_\phi = 0$ at fluid domination fixed point $\mathcal{F}$: $x_1 = x_2 = y_1 = 0$.
- Search for evolution into dark energy domination today.

Approaches to finding transients

Cosmological initial conditions

- Start in the past at phase of matter domination.
- $\Omega_\phi = 0$ at fluid domination fixed point $\mathcal{F}$: $x_1 = x_2 = y_1 = 0$.
- Search for evolution into dark energy domination today.

Observable initial conditions

- Start "today" with observable parameters $\omega_\phi \sim -1$, $\Omega_\phi \sim 0.7$.
- Seems underdetermined, but fixes i.c. to $x_1 = x_2 = 0$, $y_1 = \sqrt{0.7}$.
- Integrate backwards to determine past evolution.
- Trajectory is always viable today, but past has to be explained.

Stringy models I: universal moduli

$$S = \int d^4x \sqrt{-g} \left(\frac{{M_p}^2}{2} R - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2} f(\phi_1)^2 (\partial\phi_2)^2 - V(\phi_1) \right)$$

Kähler moduli T in IIB flux vacua:

With $T = \tau + i\vartheta$, $K = -3\log(T + \overline{T})$:

$$\phi_1 = \sqrt{3/2}\,\log(\tau)\,, \qquad \phi_2 = \vartheta\,;$$

$$f(\phi_1) = \sqrt{3/2}\,e^{\sqrt{3/2}\,\phi_1}\,, \quad V(\phi_1) \sim \frac{1}{\tau} = e^{-\sqrt{3/2}\,\phi_1}$$

$$\Rightarrow \quad k_1 = \sqrt{\frac{2}{3}}\,, \qquad k_2 = \sqrt{6}$$

Stringy models I: universal moduli

Kähler moduli T in IIB flux vacua:

With $T = \tau + i\vartheta$, $K = -3\log(T + \overline{T})$:

$$k_1 = \sqrt{\frac{2}{3}}\,, \qquad k_2 = \sqrt{6}$$

Kähler potential for chiral superfield X: $K = -p\log(X + \overline{X})$

$$k_1 = \sqrt{\frac{2}{p}}\,, \qquad k_2 = \sqrt{2p}$$

Stringy models I: universal moduli

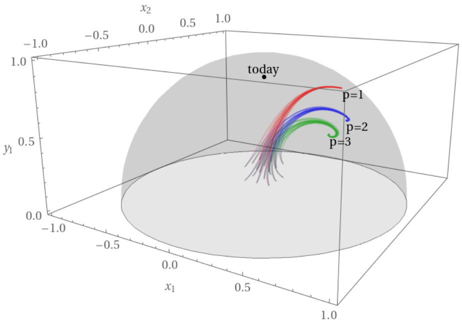
Kähler potential for chiral superfield X: $K = -p \log(X + \overline{X})$

$$k_1 = \sqrt{\frac{2}{p}}, \qquad k_2 = \sqrt{2p}$$

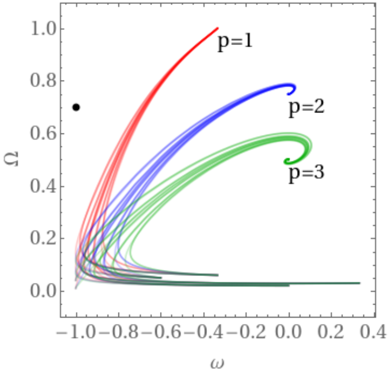
p	X	Theory	Sources	$\mathcal{M}_{\mathrm{internal}}$
1	$S = e^{-\varphi} + \mathrm{i}\, a$	Heterotic	—	SU(3) str.
2	$T_2 = \mathrm{Vol}(\Sigma_4^{(2)}) + \mathrm{i} \int_{\Sigma_4^{(2)}} C_{(4)}$	Type IIB	D3/D7, O3/O7	K3-fibered CY_3
3	$T = \mathrm{Vol}(\Sigma_4) + \mathrm{i} \int_{\Sigma_4} C_{(4)}$	Type IIB	D3/O3	CY_3
...				
7	$Z = \mathrm{Vol}(\Sigma_3) + \mathrm{i} \int_{\Sigma_3} A_{(3)}$	M-theory	KK6/KKO6	G_2 str.

Stringy models I: universal moduli

Approach 1: Starting from matter domination

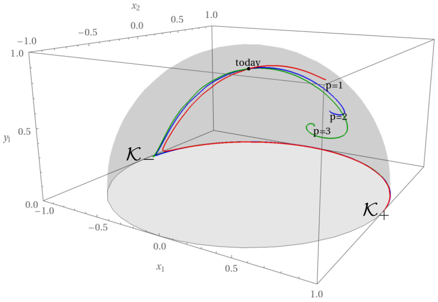


No viable trajectories starting from the matter dominated fixed point $\mathcal{F}$.

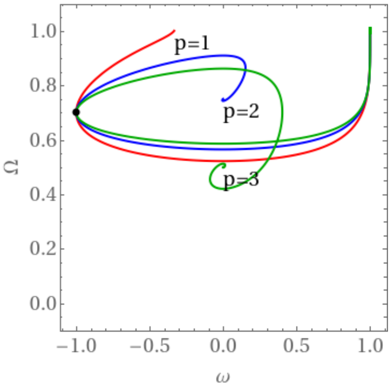


Stringy models I: universal moduli

Approach 2: Starting from observed parameters

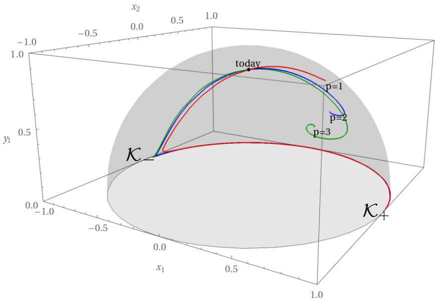


Trajectories passing through the observed point.

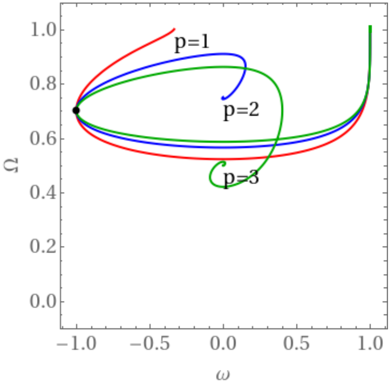


Stringy models I: universal moduli

Approach 2: Starting from observed parameters



Trajectories passing through the observed point.

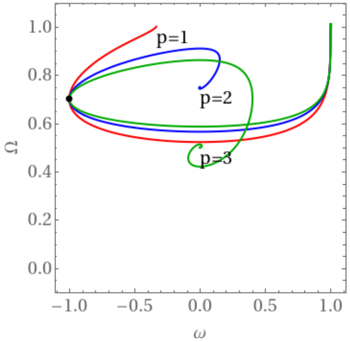


- Universal "spine" asymptoting to $\mathcal{K}_\pm$ fixed points (in the past).
- No trajectories come close to matter domination $\Omega_\phi = 0$.

Stringy models I: universal moduli

What is matter domination really?

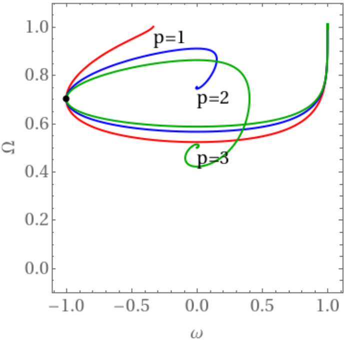
- Def. matter: $\omega_m = 0$
- Is $\Omega_\phi \neq 0$ matter dominated, as long as $\omega_\phi = 0$?
- Trajectories have $\omega_\phi = 0$ in past.



Stringy models I: universal moduli

What is matter domination really?

- Def. matter: $\omega_m = 0$
- Is $\Omega_\phi \neq 0$ matter dominated, as long as $\omega_\phi = 0$?
- Trajectories have $\omega_\phi = 0$ in past.



Initial conditions are harder to justify, but maybe not impossible!

Stringy models II: blow-up modes

Non-universal blow-up modes

- Govern the size of a blow-up (singularity resolution)
- Weak swiss cheese type: $\mathcal{V} = \alpha\,{\tau_b}^{3/2} - \lambda\,{\tau_s}^{3/2}$
- For $\tau_b \gg \tau_s \gg 1$: Kähler potential is power law, not logarithm!

$$K = -3\ln\tau_b + 2\left(\frac{\tau_s}{\tau_b}\right)^{\frac{3}{2}}$$

Stringy models II: blow-up modes

Non-universal blow-up modes

- Govern the size of a blow-up (singularity resolution)
- Weak swiss cheese type: $\mathcal{V} = \alpha\,{\tau_b}^{3/2} - \lambda\,{\tau_s}^{3/2}$
- For $\tau_b \gg \tau_s \gg 1$: Kähler potential is power law, not logarithm!

Kinetic terms for canonically normalized blow-up mode

$$\mathcal{L}_{\rm kin} = -\frac{1}{2}\sqrt{-g}\left((\partial\phi_1)^2 + \left(\frac{M_p}{\phi_1}\right)^{2/3}(\partial\phi_2)^2\right)$$

with $\phi_1 \sim (\frac{\tau_s}{\tau_b})^{\frac{3}{4}}$ and $\phi_2 \sim (\frac{\vartheta_s}{i\tau_b})^{\frac{3}{4}}$ the corresponding axion.

$$f(\phi_1) = \left(\frac{M_p}{\phi_1}\right)^{1/3} \quad \Rightarrow \quad k_1 = \frac{1}{3}\frac{M_p}{\phi_1}\,.$$

Stringy models II: blow-up modes

Kinetic terms

$$f(\phi_1) = \left(\frac{M_p}{\phi_1}\right)^{1/3} \quad \Rightarrow \quad k_1 = \frac{1}{3}\frac{M_p}{\phi_1}\,.$$

Potential terms

- Perturbative corrections (string loops, higher derivative effects).
- Depends on the explicit setup, but is always power-law:

$$V(\phi_1) = V_0\left(\frac{M_p}{\phi_1}\right)^{\pm 2/3} \quad \text{or} \quad V(\phi_1) = \frac{V_0}{C - (\phi_1/M_p)^{2/3}}\,.$$

$$\Rightarrow \quad k_2 \sim \frac{2}{3}\frac{M_p}{\phi_1}\,.$$

Stringy models II: blow-up modes

General power-law models [Cicoli/Dibitetto/Pedro '20]

Power-law Kähler and scalar potentials fall into class of models

$$f(\phi_1) = \left(\frac{M_p}{\phi_1}\right)^{p_1} \quad \text{and} \quad V(\phi_1) = V_0 \left(\frac{M_p}{\phi_1}\right)^{p_2}$$

$$\text{with} \qquad k_1 = p_1 \frac{M_p}{\phi_1}, \quad k_2 = p_2 \frac{M_p}{\phi_1}.$$

Hierarchy unnatural?

- Viable trajectories need at least $\mathcal{O}(10)$ hierarchy between p_1, p_2.
- In blow-up example, $p_1/p_2 = 1/2 \ll \mathcal{O}(10)$
- Other string examples also fail to produce hierarchy.

Conclusions

- Many accelerating models satisfy the dS conjecture.
- But can they model our late time cosmology?
- Multi-field approach natural in string theory.
- Kinetic coupling allows for a steep potential.
- Fixed points don't include our universe. Transients!

Conclusions

- Many accelerating models satisfy the dS conjecture.
- But can they model our late time cosmology?
- Multi-field approach natural in string theory.
- Kinetic coupling allows for a steep potential.
- Fixed points don't include our universe. Transients!

- Universal moduli have viable transients with questionable past.
- Non-universal moduli can't generate necessary $p1/p2$ hierarchy.

Thank you for your attention!

Autonomous system

The autonomous system:

$$x_1' = 3x_1(x_1^2 + x_2^2 - 1) + \sqrt{\frac{3}{2}}(-2k_1 x_2^2 + k_2 y_1^2) - \frac{3}{2}\gamma x_1(x_1^2 + x_2^2 + y_1^2 - 1)\,,$$

$$x_2' = 3x_2\left(x_1^2 + x_2^2 - 1\right) + \sqrt{6}k_1 x_1 x_2 - \frac{3}{2}\gamma x_2\left(x_1^2 + x_2^2 + y_1^2 - 1\right),$$

$$y_1' = -\sqrt{\frac{3}{2}}k_2 x_1 y_1 - \frac{3}{2}\gamma y_1\left(x_1^2 + x_2^2 + y_1^2 - 1\right) + 3y_1\left(x_1^2 + x_2^2\right),$$

and the cosmological parameters:

$$\Omega_\phi' = -3\left(\Omega_\phi - 1\right)\Omega_\phi(\omega_b - \omega_\phi)\,,$$

$$\omega_\phi' = (\omega_\phi - 1)\left(-k_2\sqrt{3(\omega_\phi + 1)\Omega_\phi - 6x_2^2} + 3(1 + \omega_\phi)\right).$$

Conserved currents and Q-balls

Dynamic Q-ball formation

- Our models have a conserved current $J^\mu = \sqrt{-g} f^2 \partial^\mu \phi_2$.
- In such models Q-balls may appear. [Coleman '85; Krippendorf/Muia/Quevedo '18]
- Production of Q-balls screens the dark energy. [Kasuya '01; Li/Hao/Liu '01]
- Must ensure that our models avoid producing Q-balls!

Conserved currents and Q-balls

Dynamic Q-ball formation

- Our models have a conserved current $J^\mu = \sqrt{-g} f^2 \partial^\mu \phi_2$.
- In such models Q-balls may appear. [Coleman '85; Krippendorf/Muia/Quevedo '18]
- Production of Q-balls screens the dark energy. [Kasuya '01; Li/Hao/Liu '01]
- Must ensure that our models avoid producing Q-balls!

Jeans length

Q-balls can only form if

$$0 < \frac{k^2}{a^2} < 3H^2 M_p^2 \left(\left(4k_1^2 - 2k_1'\right) x_2^2 - \left(k_2^2 - k_2'\right) y_1^2 \right) ,$$

leaving a safe wedge in the center of parameter space.

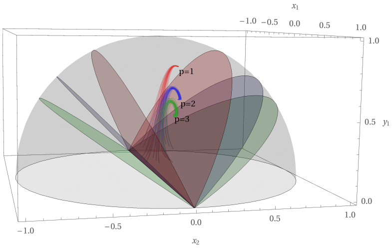
Conserved currents and Q-balls

Jeans length for constant k_i

Q-balls can only form if

$$0 < \frac{k^2}{a^2} < 3H^2 M_p^2 \left(4k_1^2\, x_2^2 - k_2^2\, y_1^2\right) ,$$

leaving a safe wedge in the center of parameter space.



Implications

- No problem for constant k_i or power-law models.
- May become important if a model has $k1/k2$ hierarchy.